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## Third Semester B.E. Degree Examination, July/August 2022 Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 80

**Note: Answer any FIVE full questions, choosing ONE full question from each module.**

### Module-1

- 1 a. Find the Fourier series expansion of  $f(x) = 2x - x^2$  in  $(0, 3)$ . (08 Marks)  
 b. The turning moment T is given for a series of values of the Crank angle  $\theta^\circ = 75^\circ$

$\theta^\circ$	0	30	60	90	120	150	180
T	0	5224	8097	7850	5499	2626	0

Obtain the first four terms in a series of sines to represent T. Also calculate T for  $\theta = 75^\circ$ . (08 Marks)

**OR**

- 2 a. Obtain Fourier series for the function  $f(x)$  given by  

$$f(x) = \begin{cases} \pi + x & -\pi < x < 0 \\ \pi - x & 0 \leq x < \pi \end{cases}$$
 Hence deduce  $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ . (08 Marks)  
 b. i) Define Half range Fourier sine series of  $f(x)$ . (02 Marks)  
 ii) Find the half range Cosine series of  $f(x) = x^2$  in the range  $0 \leq x \leq \pi$ . (06 Marks)

### Module-2

- 3 a. Find the Fourier transform of  $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ . Hence evaluate  $\int_0^{\infty} \left( \frac{\sin x}{x} \right) dx$ . (06 Marks)  
 b. Find the inverse sine transform of  $F_s(\alpha) = \begin{cases} 1 & 0 \leq \alpha < 1 \\ 2 & 1 \leq \alpha < 2 \\ 0 & \alpha \geq 2 \end{cases}$  (05 Marks)  
 c. Find the inverse Z- transform of  $\frac{2z^2 + 3z}{(z+2)(z-4)}$ . (05 Marks)

**OR**

- 4 a. Find the Fourier Sine transform of  $f(x) = e^{-|x|}$  and hence show that  

$$\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}, m > 0.$$
 (06 Marks)  
 b. Find the Z-transform of i)  $\text{Coshn}\theta$  ii)  $\text{Sinhn}\theta$ . (05 Marks)  
 c. Using the Z- transform, solve  $u_{n+2} + u_n = 0$  given  $u_0 = 1, u_1 = 2$ . (05 Marks)

### Module-3

- 5 a. Find the correlation coefficient between x and y
- |   |   |   |   |   |    |
|---|---|---|---|---|----|
| x | 2 | 4 | 6 | 8 | 10 |
| y | 5 | 7 | 9 | 8 | 11 |

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8 = 50, will be treated as malpractice.

- b. Fit the curve of the form  $y = a + bx + cx^2$  to the following data :

x	0	1	2	3	4
y	-4	-1	4	11	20

(05 Marks)

- c. Find the root of the equation  $2x - \log_e x = 7$  using Regula-Falsi method. Carry out 3 iteration. (05 Marks)

OR

- 6 a. If  $\theta$  is the angle between the two regression lines, show that  $\tan \theta = \left( \frac{1-r^2}{r} \right) \left[ \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \right]$ .

Explain the significance when  $r = 0$  and  $r = \pm 1$ .

(06 Marks)

- b. Use the method of least squares fit a curve of the form  $y = a e^{bx}$  for the following data :

x	0	2	4	6	8
y	150	63	28	12	5.6

(05 Marks)

- c. Find the real root of the equation  $x^4 - x = 10$  by using Newton-Raphson method, carryout 3 iteration. (05 Marks)

**Module-4**

- 7 a. Find  $f(x)$ , using Newton's interpolation formula

x	0	1	2	3	4
f(x)	-5	-10	-9	4	35

(06 Marks)

- b. Find  $f(g)$  : Using Newton's divided difference formula

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

(05 Marks)

- c. Evaluate, using Simpson's  $\left(\frac{1}{3}\right)^{\text{rd}}$  rule for  $\int_0^{\pi/2} \sqrt{\sin x} dx$  by taking 6 intervals. (05 Marks)

OR

- 8 a. A curve passing through the points (0, 18) (1, 10) (3, -18) and (6, 90). Find  $f(x)$ , using Lagrange's interpolation formula. (05 Marks)

- b. Evaluate, using Weddle's rule  $\int_0^6 \frac{e^x}{1+x} dx$  by taking 7 ordinates. (05 Marks)

- c. The area 'A' of a circle of diameter 'd' is given for the following values

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

Calculate the area of a circle of diameter 105.

(06 Marks)

**Module-5**

- 9 a. By using Green's theorem, evaluate  $\int_C [(y - \sin x)dx + \cos x dy]$  where C is the plane triangle enclosed by the lines  $y = 0$  ;  $x = \frac{\pi}{2}$  and  $y = \frac{2}{\pi}x$ . (06 Marks)
- b. Apply Stoke's theorem evaluate  $\int_C (x + y)dx + (2x - z)dy + (y + z)dz$  where C is the boundary of the triangle with vertices (2, 0, 0) (0, 3, 0) and (0, 0,6). (05 Marks)
- c. Find the curve on which the functional  $\int_0^1 (y')^2 + 12xy dx$  with  $y(0) = 0$  and  $y(1) = 1$  can be extremized. (05 Marks)

**OR**

- 10 a. Derive the Euler's equation in the form  $\frac{\partial f}{\partial y} - \frac{d}{dx} \left( \frac{\partial f}{\partial y'} \right) = 0$ . (06 Marks)
- b. Show that the geodesics on a plane are straight lines. (05 Marks)
- c. Evaluate  $\iint_S \vec{F} \cdot \hat{n} ds$  where  $\vec{F} = 4xz \hat{i} + y^2 \hat{j} + yz \hat{k}$  and S in the surface of the cube bounded by the planes  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ . (05 Marks)

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